2015 SEG INTERNATIONAL EXPOSITION AND 85TH ANNUAL MEETING NEW ORLEANS

A FIRST COMPARISON OF THE INVERSE SCATTERING SERIES NON-LINEAR INVERSION AND THE ITERATIVE LINAER INVERSION FOR PARAMETER ESTIMATION

Jinlong Yang and Arthur B. Weglein*
M-OSRP, University of Houston
October 22, 2015

Outline

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

1. Introduction and motivation
2. Inverse scattering series non-linear inversion
3. Iterative linear inversion
4. Numerical example
5. Conclusion

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Introduction and motivation

- Modeling
- Inversion

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Introduction and motivation

- Direct

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Indirect

Search for x such that $(ax^2 + bx + c)^2$ is a minimum.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Indicators of "indirect"

- model matching
- objective/cost functions
- search algorithm
- iterative linear "inversion"
- necessary and not sufficient conditions, e.g., CIG flatness

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Introduction and motivation

- There's a role for direct and indirect methods in practical real world application.
- For a normal incidence plane wave on a 1D earth, a closed-form direct inverse solution exists (e.g., Ware and Aki, (1968) JASA)

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Direct forward and direct inverse for a multi-dimensional subsurface

$$L_0 G_0 = \quad \text{and} \quad LG =$$

$$V = L_0 - L \quad \text{and} \quad {}_s = G - G_0$$

Relationship:

$$G = G_0 + G_0 V G$$

An operator identity that follows from

$$L^{-1} = L_0^{-1} + L_0^{-1}(L_0 - L)L^{-1}$$

Modeling:

$$L \to G \quad \text{and} \quad L_0; V \to G$$

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Direct forward and direct inverse

Relationship:

$$G = G_0 + G_0 V G$$

Modeling:

$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots$$

Form:

$$G - G_0 = S = ar + ar^2 + \cdots = \frac{ar}{1 - r}$$

With $a = G_0$ and $r = V G_0$

$$S = S_1 + S_2 + S_3 + \cdots$$

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Direct forward and direct inverse

$$S = S_1 + S_2 + S_3 + \cdots$$

$$S = \frac{ar}{1 - r}$$

Solve for r

$$r = \frac{S/a}{1 + S/a} = S/a - (S/a)^2 + (S/a)^3 + \cdots$$
$$= r_1 + r_2 + r_3 + \cdots$$

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Direct forward and direct inverse

$$
\begin{aligned}
r_1 &= S{=}a \\
r_2 &= -(S{=}a)^2 \\
r_3 &= +(S{=}a)^3 \\
&\vdots
\end{aligned}
$$

Each term in the inverse series that inverts

$$S = ar + ar^2 + \cdots = \frac{ar}{1-r} \quad \text{for } r$$

is computed directly in terms of S and a.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Direct forward and direct inverse

When we consider the generalized Geometric Series

$$S = \quad _s = \underbrace{G_0}_{a}\underbrace{VG_0}_{r} + \underbrace{G_0}_{a}\underbrace{\underbrace{VG_0}_{r}\underbrace{VG_0}_{r}}_{r^2} + \cdots$$

the inverse Geometric Series for

$$V = V_1 + V_2 + V_3 + \cdots = r_1 + r_2 + r_3 + \cdots$$

where each term is computed directly in terms of a 'G_0' and S (' ' the recorded data).

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Direct forward and direct inverse

$$
\begin{aligned}
G_0 V_1 G_0 &= D \\
G_0 V_2 G_0 &= -G_0 V_1 G_0 V_1 G_0 \\
G_0 V_3 G_0 &= -G_0 V_1 G_0 V_1 G_0 V_1 G_0 - G_0 V_1 G_0 V_2 G_0 - G_0 V_2 G_0 V_1 G_0 \\
&\vdots
\end{aligned}
$$

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Direct forward and direct inverse

Hence, the inverse scattering series is not only the only direct multidimensional inverse seismic solution, in addition, every term in the solution is directly computed in terms of a single unchanged reference and the recorded reflection data.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Isolated task subseries of the ISS

- If we imagine that the inverse machine that inputs reflection data and outputs earth properties needs to achieve certain tasks, then we can seek to locate terms within the general ISS that achieve those specific objectives.
- Those tasks are:
  1. Free surface multiple removal
  2. Internal multiple removal
  3. Depth imaging
  4. Parameter estimation
  5. Q compensation without knowing or needing Q

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Isolated task subseries of the ISS

- Each of these tasks has a subseries that doesn't require subsurface information.
- In addition, several of these tasks(and corresponding subseries) are model type independent. That is, there is one unchanged algorithm to achieve that task independent of whether you assume the earth is acoustic, elastic, anisotropic or inelastic.
- If anyone needed a way to see how different the ISS direct inverse is different from linear updating and model matching. It is not conceivable to perform model type independent model matching and updating, e.g., for multiple removal.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Non-linearity (2 types)

- Innate or intrinsic non-linearity
  e.g., R.C. and material property changes.
- Circumstantial non-linearity
  – Removing multiples or depth imaging primaries.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

- The inverse scattering series is (once again) unique in its ability to directly address either alone, let alone these two in combination.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

- The first term in a task specific subseries that addresses a potential circumstantial non-linearity first decides if its assistance and contribution is required in your data.
- And only if decides that it is, then it starts to compute if there is no nonlinear circumstantial issue to address, it computes an integrand which is zero.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

- The latter is a very sophisticated and impressive (and surprising) quality of task specific subseries
- We call that property purposeful perturbation theory.
- It decides if its assistance is needed before it acts!

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

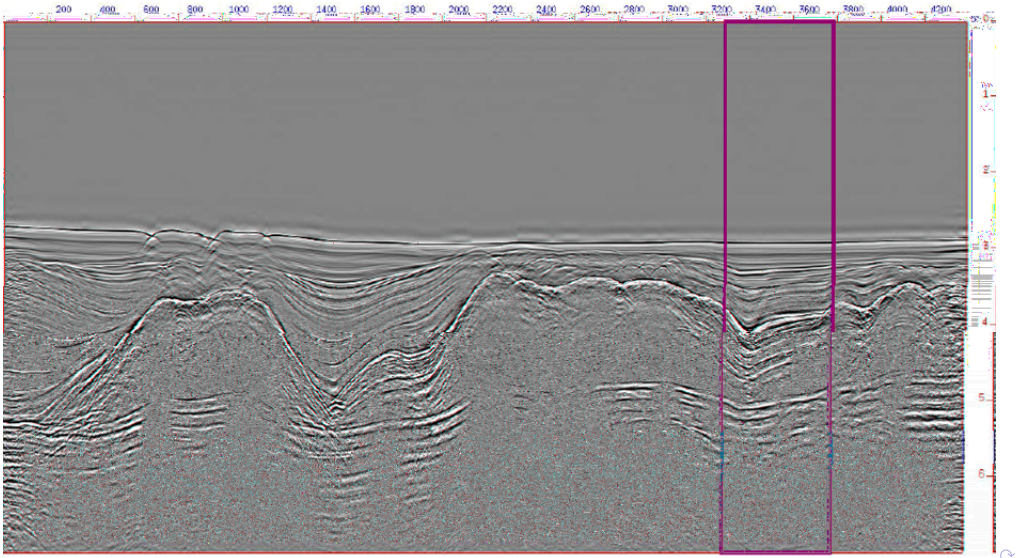
Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Offshore Brazil data: Before internal multiple attenuation



Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

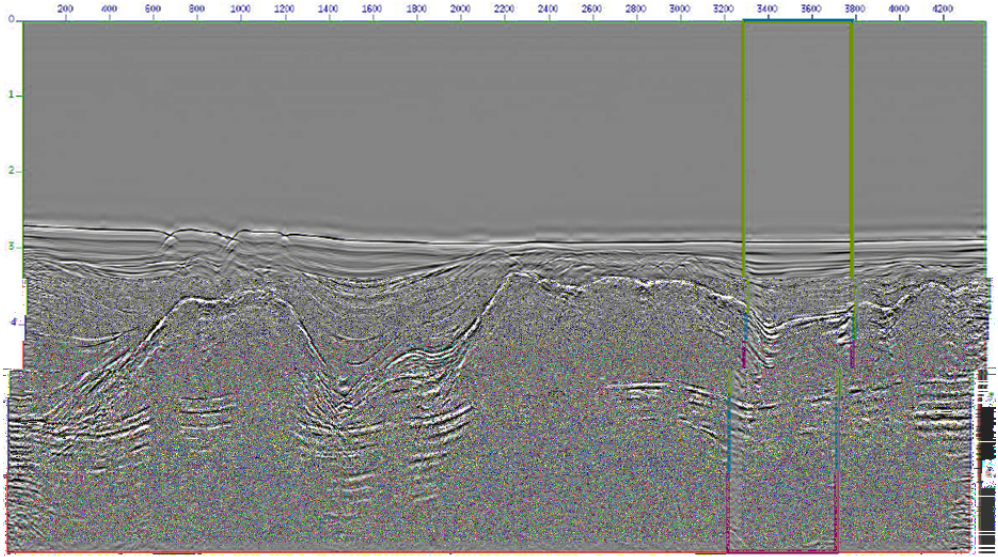
Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Offshore Brazil data: After internal multiple attenuation



2D Elastic (isotropic)

$$L\vec{u} = \begin{pmatrix} f_x \\ f_z \end{pmatrix} \quad \text{and} \quad \hat{L}\begin{pmatrix} P \\ S \end{pmatrix} = \begin{pmatrix} F^P \\ F^S \end{pmatrix};$$

where

$$L = \left[\rho\omega^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \partial_x(\lambda+2\mu)\partial_x + \partial_z\mu\partial_z & \partial_x\lambda\partial_z + \partial_z\mu\partial_x \\ \partial_z\lambda\partial_x + \partial_x\mu\partial_z & \partial_z(\lambda+2\mu)\partial_z + \partial_x\mu\partial_x \end{pmatrix} \right];$$

$$L_0 = \left[\rho_0\omega^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} (\lambda_0+2\mu_0)\partial_x^2 + \mu_0\partial_z^2 & (\lambda_0+\mu_0)\partial_x\partial_z \\ (\lambda_0+\mu_0)\partial_x\partial_z & \mu_0\partial_x^2 + (\lambda_0+2\mu_0)\partial_z^2 \end{pmatrix} \right];$$

$$V \equiv L_0 - L = \begin{bmatrix} a_\rho\rho\omega^2 + \lambda_0 a_\gamma\partial_x + \mu_0 a_\mu\partial_z & \partial_x(\lambda_0 a_\gamma - 2\mu_0 a_\mu)\partial_z + \mu_0 a_\mu\partial_x \\ \partial_z(\lambda_0 a_\gamma - 2\mu_0 a_\mu)\partial_x + \mu_0 a_\mu\partial_z & a_\rho\rho\omega^2 + \mu_0 a_\gamma\partial_z + \lambda_0 a_\mu\partial_x \end{bmatrix};$$

with $a_\rho \equiv \frac{\rho}{\rho_0} - 1$, $a_\gamma \equiv \frac{\lambda}{\lambda_0} - 1$, $a_\mu \equiv \frac{\mu}{\mu_0} - 1$.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

The forward problem

For the elastic model, the data D is a matrix and the perturbation V is also a matrix.

$$D = \hat{G} - \hat{G}_0 = \hat{G}_0 \hat{V} \hat{G}_0 + \hat{G}_0 \hat{V} \hat{G}_0 \hat{V} \hat{G}_0 + :::$$

$$\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix} = \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} + \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \times \begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} + :::$$

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

The inverse problem (solving for r in terms of S)

$$\hat{V}^{PP} = \hat{V}_1^{PP} + \hat{V}_2^{PP} + \hat{V}_3^{PP} + \cdots$$

$$\hat{V}^{PS} = \hat{V}_1^{PS} + \hat{V}_2^{PS} + \hat{V}_3^{PS} + \cdots$$

$$\hat{V}^{SP} = \hat{V}_1^{SP} + \hat{V}_2^{SP} + \hat{V}_3^{SP} + \cdots$$

$$\hat{V}^{SS} = \hat{V}_1^{SS} + \hat{V}_2^{SS} + \hat{V}_3^{SS} + \cdots$$

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

The inverse problem (solving for r in terms of S)

$$\hat{V}^{PP} = -\nabla^2 a_\gamma - \frac{\omega^2}{\alpha_0^2}(a_\rho \partial_x^2 + \partial_z a_\rho \partial_z)\frac{1}{\nabla^2} - \left[-2\partial_z^2 a_{mu}\partial_x^2 - 2\partial_x^2 a_{mu}\partial_z^2 + 4\partial_x^2 \partial_z a_{mu}\partial_z\right]\frac{1}{\nabla^2}$$

$$\hat{V}^{PS} = \frac{\alpha_0^2}{\beta_0^2}\left[\frac{\omega^2}{\alpha_0^2}(\partial_x a_\rho \partial_z - \partial_z a_\rho \partial_x) + 2\partial_x \partial_z a_{mu}(\partial_z^2 - \partial_x^2) - 2(\partial_z^2 - \partial_x^2)a_{mu}\partial_z \partial_x\right]\frac{1}{\nabla^2}$$

$$\hat{V}^{SP} = -\left[\frac{\omega^2}{\alpha_0^2}(\partial_x a_\rho \partial_z - \partial_z a_\rho \partial_x) + 2\partial_x \partial_z a_{mu}(\partial_z^2 - \partial_x^2) - 2(\partial_z^2 - \partial_x^2)a_{mu}\partial_z \partial_x\right]\frac{1}{\nabla^2}$$

$$\hat{V}^{SS} = -\frac{\alpha_0^2}{\beta_0^2}\left[\frac{\omega^2}{\alpha_0^2}(a_\rho \partial_x^2 + \partial_z a_\rho \partial_z) + (\partial_z^2 - \partial_x^2)a_{mu}(\partial_z^2 - \partial_x^2) + 4\partial_x \partial_z a_{mu}\partial_x \partial_z\right]\frac{1}{\nabla^2}$$

$$a_{mu} = \frac{\mu - \mu_0}{\gamma_0} = \frac{\beta_0^2}{\alpha_0^2}a_\mu$$

The inversion equation,

$$
\begin{aligned}
&G_0 V_1 G_0 = D \\
&G_0 V_2 G_0 = -G_0 V_1 G_0 V_1 G_0 \\
&G_0 V_3 G_0 = -G_0 V_1 G_0 V_1 G_0 V_1 G_0 - G_0 V_1 G_0 V_2 G_0 - G_0 V_2 G_0 V_1 G_0 \\
&\qquad \vdots
\end{aligned}
$$

$$
\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix} = \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix};
$$

$$
\begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_2^{PP} & \hat{V}_2^{PS} \\ \hat{V}_2^{SP} & \hat{V}_2^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}
$$

$$
= -\begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}
$$

$$
\times \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix};
$$

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Hence, for $\hat{V}^{PP} = \hat{V}_1^{PP} + \hat{V}_2^{PP} + \hat{V}_3^{PP} + :::$ any one of the four matrix elements of V requires $\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix}$.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Direct forward and direct inverse

- The relationship $G = G_0 + G_0VG_0 + G_0VG_0VG_0 + \cdots$ provides a Geometric forward series rather than a Taylor series.
- In general, a Taylor series doesnt have an inverse series; however, a Geometric series has an inverse series.
- All conventional current mainstream inversion, including iterative linear inversion and FWI, are based on a Taylor series concept.
- Solving a forward problem in an inverse sense is not the same as solving an inverse problem directly.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Solving the forward problem in an inverse sense,

$$S = ar + ar^2 + \cdots + ar^n$$

$$S - ar - ar^2 - \cdots - ar^n = 0$$

n roots, $r_1; r_2; \cdots; r_n$

As $n \rightarrow infty$, number of roots $\rightarrow infty$

However, from $S = \frac{ar}{1-r}$, we found the direct inverse $r = \frac{S}{a+S}$ (one real root).

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Introduction and motivation

- For the elastic case, there are many ways for iterative to go off track.
- In this talk, we will focus on the parameter estimation subseries and reduce it to a 1D normal incidence wave on a 1D acoustic earth where a single measured pressure wave is the input data.
- Under that very limited and focused circumstance, we are examining the difference between the iterative linear inverse and the direct inverse represented by the ISS parameter estimation subseries.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Introduction and motivation

- A direct comparison is realizable in this specific example. The iterative approach shown in this example doesn't incorporate practical issues, e.g., the numerical noise and the different generalized inverses at each step. However, the ISS method performs as it does in practice with an analytic and unchanged inverse at every step.

- We provide the iterative linear inverse method with the same analytic Frechet derivative/sensitivity matrix that derives from the inverse scattering series theory.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

ISS direct non-linear inversion

The inverse scattering series (ISS) (Weglein et al., 2003) for parameter estimation (property determination) follows from the ISS equations

$$\begin{aligned} D &= [G_0 V_1 G_0]_{ms} && (1) \\ 0 &= [G_0 V_2 G_0]_{ms} + [G_0 V_1 G_0 V_1 G_0]_{ms} && (2) \\ &\vdots \end{aligned}$$

where D is $G - G_0$ on the measurement surface and V_1 is first-order in data. The inverse scattering series provides a direct method for obtaining subsurface information by inverting the series order-by-order to solve for the perturbation operator V, using only the measured data D and a reference wave field G_0.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Iterative linear inversion

The iterative linear inversion starts with a linear approximation of a Taylor series,

$$f(m) = f(m_0) + f'(m_0)(m - m_0); \tag{3}$$

where

- m: the actual earth model
- m_0: the reference model
- $f(m)$: the actual recorded reflection data from the earth m
- $f(m_0)$: the recorded data if the earth was model m_0

Equation (3) is an approximate linear relationship that can be solved for $(m - m_0)$ if we know $f'(m_0)$, $f(m)$ and $f(m_0)$.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

To find $f'(m_0)$, a finite difference approximation is often employed

$$f'(m_0) = \frac{f(m_0 + \Delta m) - f(m_0)}{\Delta m}; \tag{4}$$

where $m_0 + \Delta m$ is a 'nearby' model to m_0, and $f(m_0 + \Delta m)$ is the corresponding data. That process requires two modeling steps and deciding what 'direction' nearby is, and results in an approximate $f'(m_0)$. Given $f'(m_0)$, equation (3) is inverted to find a linear estimate of $(m - m_0) \equiv (\Delta M)_1$.

The next step is to update the reference m_0, by adding $(\Delta M)_1$ to m_0 finding m_0' and the process repeats

$$f(m) = f(m_0') + f'(m_0')(m - m_0') \tag{5}$$

to find $(\Delta M)_2 \equiv m - m_0'$ and is therefore called iterative linear inversion.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

The linear approximate form of the forward Taylor series (equation 3) in the language of scattering theory is

$$G = G_0 + G_0 V G_0 \tag{6}$$

and represents a linear modeling equation for G in terms of G_0 and V.

For the purpose of inversion, the relevant ISS equations are (1) and (2) $\cdots$. Equation (1) provides $D = G_0 V_1 G_0$, where $D = G - G_0$. It is the exact equation to solve for V_1, the portion of V that is linear in D.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

In general, a Taylor series doesn't have an inverse series. Hence, those depending on a Taylor series framework use a modeling equation, in a linear approximate form, as an indirect model matching inversion machine. Using equation (3) as the first step in a linear iterative procedure corresponds to equation (1). However, equation (1) provides an exact (not a finite difference approximate) Frechet derivative/sensitivity matrix computed directly in terms of the model, m_0, i.e., $G_0 G_0$.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

For a homogeneous reference medium, the inverse of $f'(m_0)$ is analytic, and in general corresponds to a constant velocity prestack Stolt FK migration. We arrange for the first linear step in the iterative procedure to benefit from the exact Frechet derivative that ISS provides from the linear inverse from equation (1). Hence, we arrange for them to agree at the first step in their respective procedures. That linear inverse agreement will not be the case when the linear iterative is actually applied in practice (see e.g. equation 4). For steps beyond linear, the ISS and iterative linear inverse are completely different. That's what we examine below.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

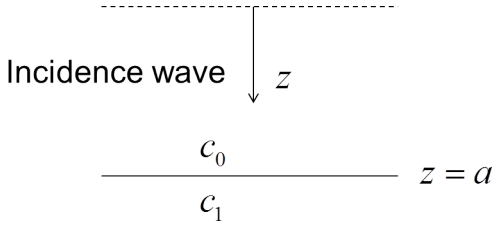
Numerical example

Conclusion

Acknowledgment

1D normal incidence wave on a 1D acoustic earth with a single reflector

Consider the example: The model consists of two half-spaces with acoustic velocities c_0 and c_1 and an interface located at $z = a$.



Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

ISS non-linear inversion in 1D one parameter case

For this problem

$$L = \frac{d^2}{dz^2} + \frac{!^2}{c^2(z)} \quad \text{and} \quad L_0 = \frac{d^2}{dz^2} + \frac{!^2}{c_0^2}; \tag{7}$$

and we characterize the velocity configuration as,

$$(z) \equiv 1 - \frac{c_0^2}{c^2(z)}: \tag{8}$$

The perturbation V is

$$V(z) = \frac{!^2}{c_0^2} \ (z); \tag{9}$$

and the inverse series for V becomes

$$(z) = \ _1(z) + \ _2(z) + \ _3(z) + \cdots : \tag{10}$$

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

ISS non-linear inversion in 1D one parameter case

For the normal incident spike wave on a 1D acoustic medium, equation (1) solves for $_1$ in terms of the single trace $D(t)$ (Shaw et al., 2004) as

$$_1(z) = 4\int_{-\infty}^{z} D(z')dz'; \tag{11}$$

where $z' = c_0 t$=2.

For a single reflector,

$$_1(z) = 4RH(z-a): \tag{12}$$

Equation (12) represents an analytic inverse of

$$D = G_0 V_1 G_0 \quad \text{or} \quad f(m) - f(m_0) = f'(m_0)\Delta m \tag{13}$$

for $_1$.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

ISS non-linear inversion in 1D one parameter case

The entire inverse series (equations (1), (2), $\cdots$) for this normal incidence plane wave on a 1D acoustic medium is presented in (S. Shaw et al., 2004 and Xu Li et al., 2011),

$$(z) = \ _1(z) - \frac{1}{2}\ _1^2(z) + \frac{3}{16}\ _1^3(z) + :::: \tag{14}$$

For a single reflector

$$= 4R - 8R^2 + 12R^3 - 16R^4 + :::: = 4R\sum_{n=0}^{\infty}(n+1)(-R)^n \tag{15}$$

(H. Zhang et al., 2006 and Xu Li et al., 2011)

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Convergence of ISS non-linear inversion

The convergence of the inverse series (equation (15)) can be tested by the ratio test (Xu Li et al., 2011),

$$\left| \frac{n+1}{n} \right| = \left| \frac{(n+2)(-R)^{n+1}}{(n+1)(-R)^{n}} \right| = \left| \frac{n+2}{n+1} R \right| : \tag{16}$$

If $\lim_{n\to\infty} \left| \frac{\alpha_{n+1}}{\alpha_n} \right| < 1$, this series converges absolutely. That is

$$|R| < \lim_{n\to\infty} \frac{n+1}{n+2} = 1: \tag{17}$$

Therefore, the ISS direct inverse subseries converges when the reflection coefficient $|R|$ is less than 1, which is always true. Hence, for this example, the ISS inverse subseries will converge under any velocity contrast between the two media.

Iterative linear inversion in 1D one parameter case

Use the first estimate of $= \ _1^1$ to compute the first estimate of $c_1 = c_1^1$. Then, choose the first estimate of $c_1 = c_0(1 - \ _1^1)^{-1/2} \equiv c_1^1$ as the new reference velocity, $c_0^1 = c_0(1 - \ _1^1)^{-1/2}$, where $\ _1^1 = 4R_1$ and $R_1 = \frac{c_1 - c_0}{c_1 + c_0}$.
Repeat the linear process with a new reflection coefficient R_2 (again exploiting the analytic inverse provided by ISS)

$$R_2 = \frac{c_1 - c_0^1}{c_1 + c_0^1}; \quad \ _1^2 = 4R_2 \text{ and } c_1^2 = c_0^1(1 - \ _1^2)^{-1/2} = c_0^2 \tag{18}$$

$$\dots \tag{19}$$

$$R_{n+1} = \frac{c_1 - c_0^n}{c_1 + c_0^n}; \quad \ _1^{n+1} = 4R_{n+1} \text{ and } c_1^n = c_0^{n-1}(1 - \ _1^n)^{-1/2} = c_0^n; \tag{20}$$

where $\ _1^n = n^{th}$ estimate of $\ _1$ and $c_1^n = n^{th}$ estimate of c_1. The questions are (1) under what conditions does c_1^n approach c_1, and (2) when it converges, what is its rate of convergence.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

ISS inversion v.s. Iterative inversion

The ISS method for always converges and the resulting can be used to find c_1. For the iterative linear inverse, there are values of $_1$ such that you cannot compute a real c_1^1. When $\frac{1}{1} > 1$ and $4R > 1$, $R > 1$=4 and you cannot compute an updated reference velocity and the method simply shuts down and fails. The inverse scattering series never computes a new reference and doesn't suffer that problem, with the series for always converging and then outputting c_1.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

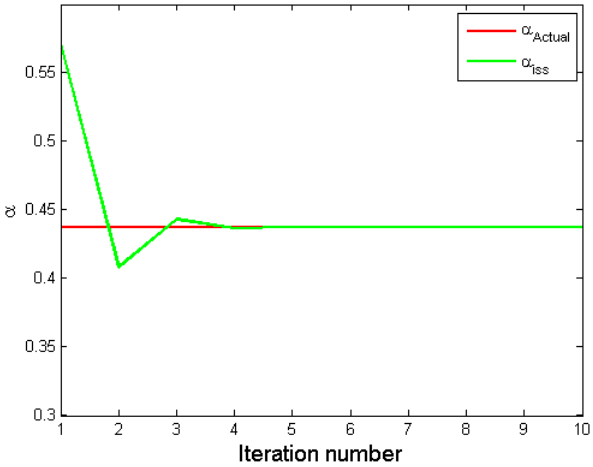
Numerical example

Conclusion

Acknowledgment

Estimated α by ISS method

$c_0 = 1500m/s,\ c_1 = 2000m/s,\ R = 0.1429,\quad = 0.4375$



Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

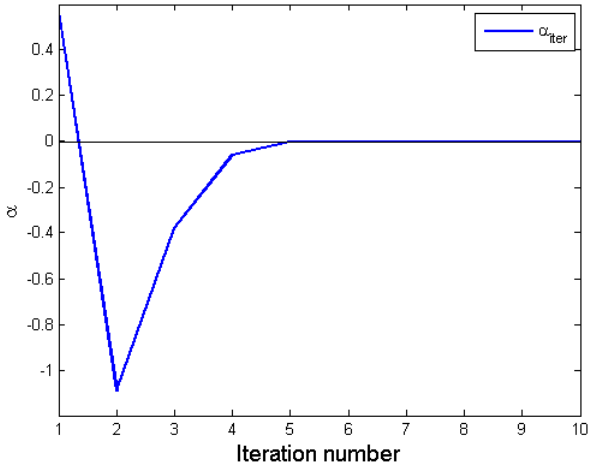
Numerical example

Conclusion

Acknowledgment

α_1^n calculated by iterative inverse

$c_0 = 1500m/s,\ c_1 = 2000m/s,\ R = 0.1429$



Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

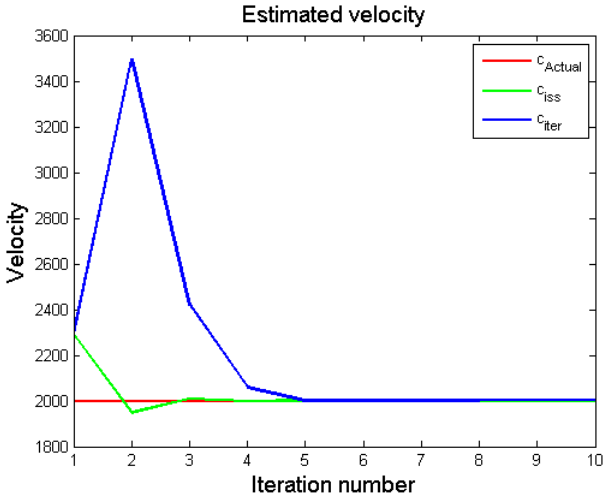
Numerical example

Conclusion

Acknowledgment

Velocity estimation

$c_0 = 1500m/s,\ c_1 = 2000m/s,\ R = 0.1429,\quad = 0.4375$



Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

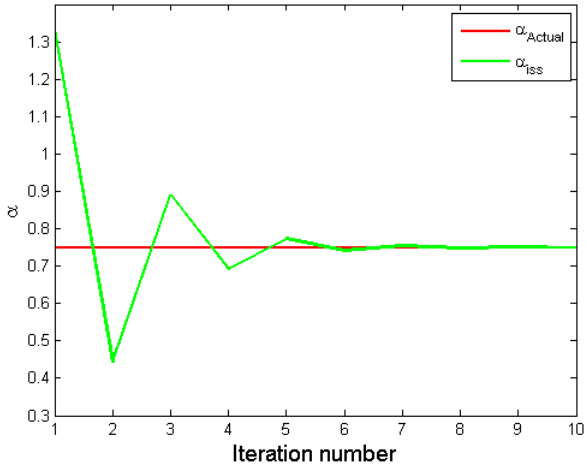
Numerical example

Conclusion

Acknowledgment

Estimated α by ISS method

$c_0 = 1500m/s,\ c_1 = 3000m/s,\ R = 0.3333,\quad = 0.7500$



Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

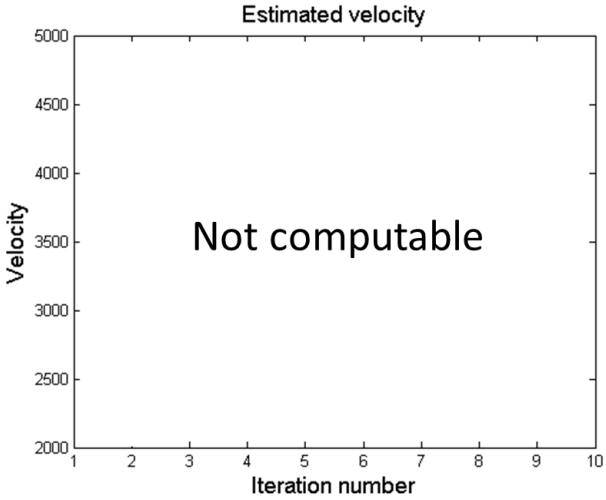
Numerical example

Conclusion

Acknowledgment

Velocity estimation by iterative inversion

$c_0 = 1500 m/s,\ c_1 = 3000 m/s,\ R = 0.3333,\quad _1 = 1.3333$



Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

Conclusion

- The ISS direct non-linear inversion and the iterative inversion are examined and compared in a 1D one parameter model with a single reflector. The ISS method always converges to the answer but the linear iterative doesn't. When they both converge, the ISS method converges faster.
- The iterative linear inversion is not equivalent to the ISS direct non-linear solution. For more complicated circumstances, the differences are greater, not just on the algorithms, but also on data requirements and on the sensitivity to the bandlimited and noisy nature of the seismic data.

Outline

Introduction and motivation

Inverse scattering series non-linear inversion

Iterative linear inversion

ISS non-linear inversion

Iterative linear inversion

Numerical example

Conclusion

Acknowledgment

ISS delivery/promise

- model-type independent algorithm
  - for free-surface multiple and internal multiples
- a framework analysis and method for direct amplitude (for AVO and FWI)
- the promise of direct depth imaging without a velocity model
  – feasibility demonstrated of Kristin field data

M-OSRP

Sponsors